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An Assessment of the Quality of Selected Finite Difference Schemes for Time Dependent Compressible Flows

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An assessment has been made of the character of the results near discontinuities yielded by selected numerical schemes of first-order (Rusanov and Van Leer schemes), second-order (Richtmyer and MacCormack schemes) and third-order (Rusanov scheme) and their capabilities are compared. The effectiveness of the Shuman Switch for use in higher order schemes has also been investigated. The Van Leer scheme is found to be preferable to the Rusanov scheme as it has a higher resolving power. The MacCormack scheme appears to behave better than the Richtmyer scheme in all aspects of shock handling. The Shuman switch is found to eliminate overshoots, undershoots and oscillations and fight nonlinear instabilities only by smearing the profiles. All schemes except the third order scheme produce artificial density profile near the wall when a shock wave is reflected from the wall.

INTRODUCTION

A number of finite-difference schemes are available for solving unsteady, inviscid, compressible flow problems. From time to time several attempts have been made to rate the various schemes according to their merit. A common evaluating procedure is to subject the schemes to a set of well-chosen test problems (for which, preferably, exact solutions are known). The results of four such attempts [1–4] are summarized in Table I. An obvious conclusion that emerges from an examination of the table is that there is no agreement among the previous investigators regarding the choice of a method. However, certain broad deductions could be made. A higher order scheme does not necessarily handle a discontinuity adequately except for perhaps the sharpness of the contact discontinuity profile. The best first-order methods are superior to second- and third-order methods in shock handling. However, a third-order method gives least error in the description of a rarefaction wave. In this paper, we shall concentrate on some schemes and properties of schemes that were not fully considered in the earlier investigations.

Specifically, we wish to address ourselves to the following questions:

(1) How does Van Leer's [5] first-order scheme (diffusion proportional to square of local Courant number) compare with Rusanov's [6] first-order scheme (diffusion proportional to local Courant number)?

(2) How does the noncentered MacCormack [7, 8] scheme compare with the Richtmyer [9] scheme?

(3) What are the effects of the Shuman Switch [10] on the various schemes?

(4) How do the various schemes behave near the wall boundary in a shock reflection problem?

(5) How well do the higher order schemes represent a standing shock?

The answers to these questions along with the conclusions available in the literature (Table I) are expected to provide more insight into the behavior of schemes and assist in the choice of a scheme for a new problem.

Governing Equations, Numerical Schemes, and Test Problems

The governing equations for one-dimensional unsteady, inviscid, compressible flows in vector form are given by

$$W_t + F_X = 0$$

where

$$W = \begin{bmatrix} \rho \\ \rho u \\ e \end{bmatrix}, \qquad F = \begin{bmatrix} \rho u \\ p + \rho u^2 \\ (e + p)u \end{bmatrix}$$

with ρ = density, u = velocity, p = pressure, e = total energy = internal energy + $\frac{1}{2}\rho u^2 = (1/(\gamma - 1))(P/\rho) + (1/2)\,\rho u^2$, and γ = ratio of specific heats. The subscripts t and X refer to partial derivatives with respect to time and space coordinates respectively.

The present study is restricted to five schemes: (1) First-order Rusanov [6] scheme, (2) First-order Van Leer [6] scheme, (3) second-order Richtmyer [9] scheme, (4) second-order MacCormack [7, 8] scheme and (5) third-order scheme formulated by Rusanov [11] and Burstein and Mirin [12]. In addition it covers the behavior of the Shuman Switch [10] when it is incorporated in higher order schemes. The Shuman Switch employs the idea of reducing the order of accuracy to that of first-order scheme near the shocks since the first-order schemes handle shocks satisfactorily without oscillations. In particular, the information on the

TABLE I

Summary of Previous Evaluations of Various Schemes

Author and date	Methods investigated	Test problems	Conclusions by the respective authors
1. Rubin and Burstein [1]	1. Lax–Wendroff 2. Richtmyer 3. modified form of (2) (all of second-order)	1. Propagating shock 2. Head-on collision of two shocks 3. Overtaking of one shock by the other	The scheme (3) is the best; gives virtually no overshoots, for all the three problems considered.
2. Emery [2]	1. Lax; first-order 2. Rusanov; first-order 3. Landshoff; first-order 4. Lax–Wendroff; second-order 5. Richtmyer; second-order	1. Propagating shock 2. Reflected shock 3. Supersonic flow over a flat faced step 4. Passage of shock over a stationary cone	Comparison based on spatial and temporal resolution, ease of coding, execution time. It is concluded that Rusanov's is preferable. Lax's method is simple to program but offers the least flexibility. The Lax–Wendroff scheme yields the best spatial and temporal resolution, but difficulties in programming this method (particularly at corners and singular points) render it useful only to those interested in numerical methods for their own sake. Rusanov's method is easily programmed, offers an adjustable parameter and yields very good resolution.

		Inviscid problems	
3. Taylor, Ndefo and Masson Ref. [3]	1. First-order Rusanov 2. First-order Godunov 3. Second-order Richtmyer 4. Second-order MacCormack 5. Third-order Rusanov	1. Rarefaction wave 2. Shock wave 3. Contact discontinuity *Viscous problems* Burger's equation for a travelling plane wave of small amplitude	Schemes were compared on the basis of integrated-errors. It is concluded that the Godunov and third order schemes demonstrate the greatest accuracy and seem to have the least amount of oscillations. The third-order Rusanov method gives rise to oscillations with shocks while the Godunov scheme produces smooth profiles. The computing time for the Godonov scheme is significantly less than that for the third-order Rusanov method.
4. Anderson [4]	1. Second-order MacCormack 2. Third-order Rusanov 3. Third-order Kutler–Warming–Lomax	1. Propagation of a double shock in one-dimensional space using Burger's equation 2. Steady supersonic flow over a two-dimensional wedge 3. Reflection of shock wave from a solid boundary	Second-order methods give acceptable results for most cases. Third-order techniques provide the best results when the Courant number varies appreciably in the computational mesh. The comparison between the behaviour of K.W.L. and Rusanov scheme is given for different problems.

TABLE II

Numerical Schemes Considered[a]

Sl. no.	Scheme	Finite difference form	Range of parameters	Stability condn.	Order of accuracy space–	Order of accuracy time	Remarks
1.	Rusanov	$W_I^{N+1} = W_I^N - (q/2)(F_{I+1}^N - F_{I-1}^N) + \frac{1}{2}(\theta_{I+\frac{1}{2}} - \theta_{I-\frac{1}{2}})$ where $\theta_{I+\frac{1}{2}} = \frac{1}{2}(\alpha_I + \alpha_{I+1})(W_{I+1} - W_I)$ and $\alpha_I = \omega \cdot \sigma((u+a)_I/(u+a)_{\max})^n$ $n = 1$	$\omega = 1.0$ $\sigma = 0.7$ to 1.0	$\sigma \leqslant 1.0$ $\sigma \leqslant \omega \leqslant 1/\sigma$	2	1	Offers an adjustable parameter ω
2.	Van Leer	(as above with $n = 2$)					
3.	Richtmyer	$W_{I+\frac{1}{2}}^{N+\frac{1}{2}} = \frac{1}{2}(W_I^N + W_{I+1}^N) - (q/2)(F_{I+1}^N - F_I^N)$ $W_I^{N+1} = W_I^N - q(F_{I+\frac{1}{2}}^{N+\frac{1}{2}} - F_{I-\frac{1}{2}}^{N+\frac{1}{2}})$ i) Without Shuman Switch* ii) With Shuman Switch, $m = 1$, $m = 2$ and in exponential form with $m = 1$, $m = 2$.	$\sigma = 0.7$, 1.0 $\chi = 1, 2$	$\sigma \leqslant 1.0$	2	2	Order of accuracy reduced to 1 wherever the switch in 'ON'

4. MacCormack	$W_I^* = W_I^N - q(F_{I+1}^N - F_I^N)$ $W_I^{N+1} = \frac{1}{2}(W_I^N + W_I^*) - (q/2)(F_I^* - F_{I-1}^*)$ Shuman Switch with $m = 1$ applied for the reflected shock	$\sigma = 0.7$, $\sigma \leqslant 1.0$ 1.0 $\chi = 1.0$, 2.0	2	2	employs noncentered differences
5. Rusanov (third-order)	$W_{I+\frac{1}{2}}^{(1)} = \frac{1}{2}(W_{I+1}^N + W_I^N) - (q/3)(F_{I+1}^N - F_I^N)$ $W_I^{(2)} = W_I^N - (2q/3)(F_{I+\frac{1}{2}}^{(1)} - F_{I-\frac{1}{2}}^{(1)})$ $W_I^{N+1} = W_I^N - (q/24)(-2F_{I+2}^N + 7F_{I-1}^N - 7F_{I-1}^N + 2F_{I-2}^N)$ $-(3q/8)(F_{I+1}^{(2)} - F_{I-1}^{(2)}) - \omega(W_{I+2}^N - 4W_{I+1}^N + 6W_I^N - 4W_{I-1}^N + W_{I-2}^N)$	$\omega = 2/24$ $\sigma = 0.7$	3	3	Based on the Runge–Kutta method for ordinary differential equations

[a] $q = \Delta T/\Delta X$; u = velocity; a = speed of sound; $\sigma = q(u + a)$; N denotes number of time steps; I indicates space mesh index.

* Shuman Switch $W = L.W_I^N$, $L.$ denotes any scheme of order $\geqslant 2$

$W_I^{N+1} = W_I + \frac{1}{4}[\theta_{I+\frac{1}{2}}(W_{I+1} - W_I) - \theta_{I-\frac{1}{2}}(W_I - W_{I-1})]$ where $\theta_{I+\frac{1}{2}} = [|P_{I+1} - P_I|/|P_{k+1} - P_k|_{\max}]^m \chi$ if $S_{I+1} - S_I \leqslant 0$, and $\theta_{I+\frac{1}{2}} = 0$ if $S_{I+1} - S_I > 0$ subject to the conditions $0.5 < \chi < 2.0$, $m \leqslant l - 1$,

l = order of accuracy of the basic scheme.

P, S could be any flow properties, for which respectively $(u + a)$ and u were employed.

Exponential form is to have $\theta_{I+\frac{1}{2}} = \chi \cdot \mathrm{Exp}[1 - (|P_{k+1} - P_k|_{\max}/|P_{I+1} - P_I|)^m]$.

behaviour of the Shuman Switch and the Van Leer scheme is meagre and therefore they have been considered here in detail.

Table II summarises the different finite-difference formulations considered in this study along with their stability criteria and orders of accuracy.

Four test problems have been considered in the investigation. They have all been chosen to be one-dimensional not only because of the availability of exact solutions but also because they have been subjected extensively to numerical testing. It is realized that the conclusions from one-dimensional study cannot be easily extended to multidimensional problems. However, a scheme which does not perform adequately for one-dimensional problems is unlikely to prove useful for multidimensional problems. The physical situations studied here are described below.

(i) A shock wave propagating along a uniform duct. This problem helps to establish the shock handling capabilities of different schemes.

(ii) Shock reflection from the closed end of a duct. This illustrates the behavior of different schemes near boundaries.

(iii) A conventional shock tube flow. The problem provides scope to examine the representation of rarefaction waves and contact discontinuities.

(iv) Example of a standing shock. This problem provides the scope for testing Shuman Switch regarding its capability to fight nonlinear instabilities associated with the problem.

For purposes of comparison with earlier investigations, a shock Mach number of 3.0 for the propagating shock problem (corresponding to the work of Emery [2]) and a diaphragm pressure ratio of 6.18 (corresponding to the work of Burstein and Mirin [12]) have been employed. For the standing shock problem a pressure ratio of 11.0 (the value employed by Van Leer [13]) is used.

The results presented in the next section correspond to 100 time steps for the propagating and reflected shock problems and to 120 time steps for the shock tube problem. Significant changes in the profiles ceased to occur much before these time steps. All of the computations reported in this paper have been carried out on an IBM 360/44 system at the Indian Institute of Science Computer Center.

Results and Discussion

A large amount of computational results have been obtained as part of the investigation. We will discuss here only those results that highlight features of the different schemes not reported in earlier investigations. The results have been grouped according to the test problems considered. Figures 1–14 present a qualita-

TABLE IIIa

Comparison of Results Obtained for Different Schemes: Propagating Shock

*S*1. no.	Numerical scheme with parameter description			Fig. No.	Over- shoot %	Under shoot %	Smearing: No. of mesh widths	Integra- ted error	Time of compu- tation
1.	Rusanov (first-Order)	$\omega = 1.0$	$\sigma = 0.7$	1	—	—	7	1.0	
			$\sigma = 1.0$	1	1.9	—	3	1.0	1.0
2.	Van Leer	$\omega = 1.0$	$\sigma = 0.7$	1	0.7	—	4	0.6	
	(first-Order)		$\sigma = 1.0$	1	12.0	—	3	1.97	1.0
3.	Richtmyer (second-Order)		$\sigma = 0.7$	2	18.0	—	4	0.94	
	(Basic)		$\sigma = 1.0$	2	12.3	—	3	1.17	1.0
	Richtmyer (second-Order)	$\sigma = 1.0$	$\chi = 1.0$	2	1.46	—	5	2.43	1.5
	(with Shuman Switch)		$\chi = 2.0$	2	0.13	—	9	4.42	
4.	MacCormack (second-Order)		$\sigma = 0.7$		9.9	—	4	0.75	
			$\sigma = 1.0$		—	—	4	1.01	1.0
5.	Rusanov (third-Order)	$\omega = 2.0/24$	$\sigma = 0.7$	3	5.28	10.0	3	0.67	1.25
			$\chi = 1.0$	3	3.0	2.0	6	1.32	
	(with Shuman Switch)		$\chi = 2.0$	3	0.7	—	9	2.72	2.0

TABLE IIIb

Comparison of Results Obtained for Different Schemes: Reflected Shock

Sl. No.	Numerical scheme with parameter description			Fig. No.	Over-shoot %	Under-shoot %	Smearing No. of mesh width	Inte-graded error	% Density depr. at the wall	No. of meshes from the wall to theor. density
1.	Rusanov (first-Order)	$\omega = 1.0$	$\sigma = 0.7$	4,8	—	—	13	1.0	−2.3	20
			$\sigma = 1.0$	4,8	—	—	10	1.0	−1.75	21
2.	Van Leer (first-Order)	$\omega = 1.0$	$\sigma = 0.7$	4,8	2.0	—	9	0.83	−1.66	13
			$\sigma = 1.0$	4,8	2.3	—	8	0.74	−1.11	13
3.	Richtmyer (second-Order)		$\sigma = 0.7$	5,8	30.8	—	3	0.83	−3.09	Steep Rise
			$\sigma = 1.0$	5,8	21.13	—	3	0.59	−4.42	Steep Rise
	Richtmyer (Second-Order)	$\sigma = 1.0$	$\chi = 1.0$	5,8	2.96	—	6	0.78	−5.8	8
	(with Shuman Switch)		$\chi = 2.0$	5,8	1.8	—	9	1.12	−6.77	11
4.	MacCormack (second-Order)		$\sigma = 0.7$	6,8	19.2	—	3	0.76	+2.15	Steep Rise
			$\sigma = 1.0$	6,8	11.69	—	3	0.46	+1.23	8
5.	Rusanov (third-Order)	$\omega = 2/24$	$\sigma = 0.7$	7,8	4.9	10.0	5	0.55	+0.17	8
	Rusanov (third-Order)	$\omega = 2/24$	$\chi = 1.0$	7,8	1.2	1.0	8	0.93	—	—
	(with Shuman Switch)	$\sigma = 0.7$	$\chi = 2.0$	7,8	—	—	13	1.74	—	—

TABLE IIIc

Comparison of Results Obtained for Different Schemes: Contact Dicontinuity

Sl. No.	Numerical scheme with parameter description			Fig. No.	Undershoot	Smearing
1.	Rusanov (first-Order)	$\omega = 1.0$	$\sigma = 0.7$	10	—	38
		$\omega = 1.0$	$\sigma = 0.7$	10	—	38
			$\sigma = 1.0$		—	45
2.	Van Leer (first-Order)	$\omega = 1.0$	$\sigma = 0.7$	10	—	35
			$\sigma = 1.0$		—	45
3.	Richtmyer (Second-Order)		$\sigma = 0.7$	11	70.5	10
	(Basic Scheme)		$\sigma = 1.0$		47.06	10
	Richtmyer (Second-Order)	$\sigma = 1.0$	$\chi = 1.0$		41.2	15
	(with Shuman Switch)		$\chi = 2.0$		41.2	20
4.	MacCormack (Second-Order)			unstable		
5.	Rusanov (third-Order)	$\omega = 2/24$	$\sigma = 0.7$	12	8.82	10
	Rusanov (third-Order)					
	(with Shuman Switch)	$\omega = 2/24$	$\chi = 1.0$	12	8.82	15
		$\sigma = 0.7$	$\chi = 2.0$	12	8.82	20

Smearing is defined in terms of the number of mesh widths over which a discontinuity spreads. In case of smooth profiles all points where the numerical value differs from the theoretical value by more than 1 per-cent are considered to add to the width of discontinuity. Wherever an overshoot or an undershoot is present, the point at which the value is maximum or minimum is considered as the head or the tail respectively of the discontinuity.

Similar considerations hold good when fixing the number of meshes from the wall to the point at which the density value is equal to the theoretical value.

Overshoot is expressed as a percentage of maximum pressure (density in the case of contact discontinuity).

Undershoot is expressed as a percentage of minimum pressure (density in the case of contact discontinuity). Integrated error $= (1/\delta)\int_0^\delta | P_a - P_{Th} | \, dx$ where P_a = the actual pressure, P_{Th} = theoretical pressure. δ = width of the discontinuity. For purposes of numerical integration all the points in the region were considered since the contribution to the error from points outside the width δ, is negligible. The integrated error for the different schemes is expressed as a ratio of the error to that for the first order Rusanov Scheme. Time of computation for the different schemes is also expressed as a ratio of time required for 20 time steps to that required by the first-order Rusanov scheme.

tive picture of the capability of different schemes in describing the main features of the test problems. Quantitative results like overshoot, undershoot, shock smearing, accuracy, and time of computation have been collected in Table III for propagating shock, reflected shock, and contact discontinuity. Reference to Table III, which also includes values of the integrated error, indicates that the integrated error across a discontinuity is dependent on the value of σ, the flow property and the problem considered. The ranking of schemes based on the propagating shock (considered by Taylor *et al.* [3]) and that based on the reflected shock (present calculations) are not the same. Similarly the ranking based on density (Taylor *et al.* [3]) and that based on pressure (present calculations) are not the same. Consequently the conclusions based entirely on the values of integrated errors for various schemes could be misleading. However, some indication can be obtained as regards the merits in employing the first-order Van Leer scheme and the demerits in employing the Shuman Switch.

It also appears that the ranking of schemes based on any particular aspect would again be misleading. There does not appear to be any other particular criterion which could be universal. Hence, all aspects of scheme behavior are considered when the comparisons are made.

(i) *Rusanov and Van Leer Schemes*

Figures 1 and 4 indicate that the Van Leer scheme yields sharper shocks (both the propagating and the reflected) compared to the first order Rusanov scheme for the same set of parameters. An overshoot results for values of σ greater than 0.7. The propagating shock profile is acceptable upto $\sigma = 0.8$. In the case of the reflected shock small oscillations exist. The profile is somewhat more smeared at the tail. The prediction of density at the wall is also improved (Table IIIb).

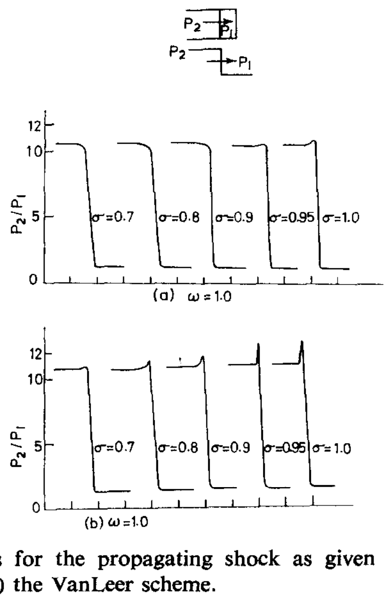


FIG. 1. Pressure profiles for the propagating shock as given by the first-order schemes: (a) the Rusanov scheme, (b) the VanLeer scheme.

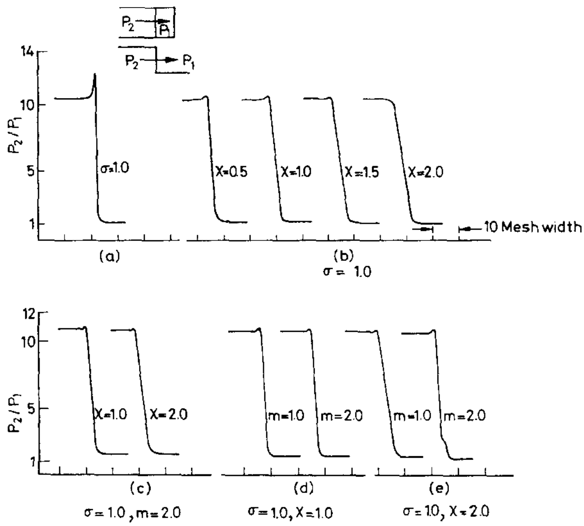


FIG. 2. Pressure profiles for the propagating shock as given by the second-order Richtmyer scheme: (a) basic form, (b) with the Shuman Switch, (c) with the Shuman Switch in quadratic form, (d) with the Shuman Switch in exponential form and $\chi = 1.0$, (e) with the Shuman Switch in exponential form and $\chi = 2.0$.

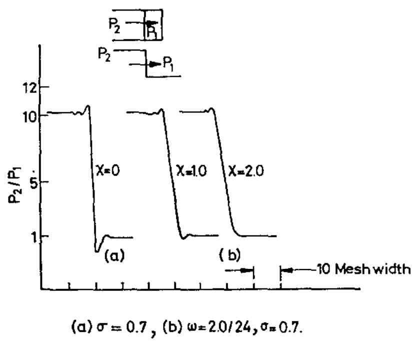
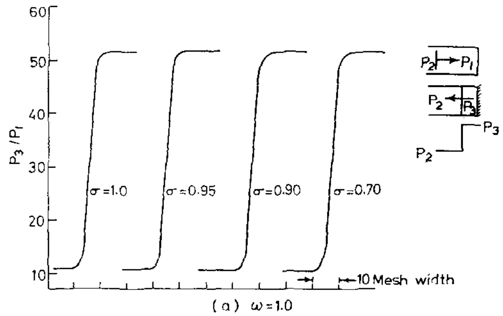


FIG. 3. Pressure profiles for the propagating shock as given by the third-order Rusanov scheme: (a) Basic form, (b) with Shuman Switch.

Regarding the shock tube flow it is found that the shock is rendered sharper but with an overshoot. The contact discontinuity behaviour is slightly improved.

(ii) *MacCormack and Richtmyer Schemes*

Studies by Taylor *et al.* [3] do not give explicit information about the MacCormack scheme. Also the form of the MacCormack scheme considered is different from that considered here. The present results show that the MacCormack



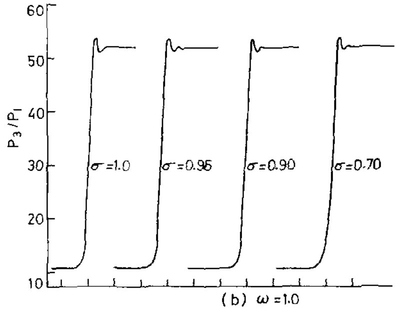


FIG. 4. Pressure profiles for the reflected shock as given by the first-order schemes: (a) the Rusanov scheme, (b) the Van Leer scheme.

scheme behaves better than the Richtmyer scheme regarding all the aspects of the propagating and the reflected shocks. But the major disadvantage is that the MacCormack scheme is unstable for the shock tube problem. Regarding the calculation of the standing shock the Richtmyer scheme was found to be unstable. The profile obtained by the MacCormack scheme, exhibiting a mild instability, was not acceptable.

(iii) *The Effects of the Shuman Switch*

Figures 2, 3, 5, 6, and 7 show that the Shuman Switch is quite effective in eliminating the oscillations, overshoots and undershoots. As is expected this is obtained at the cost of smearing of the profiles. In general, a value of $\chi = 1.0$ for both the propagating and the reflected shocks seems adequate. Some oscillations do remain for the reflected shock. Employing the value of $\chi = 2.0$ produces a cascadelike structure making the profile unacceptable. To overcome excess smearing, improved forms, namely, a value of $m = 2.0$ and the exponential form with $m = 1, 2$ were tried (see Table II). The effect of these improvements is strongly

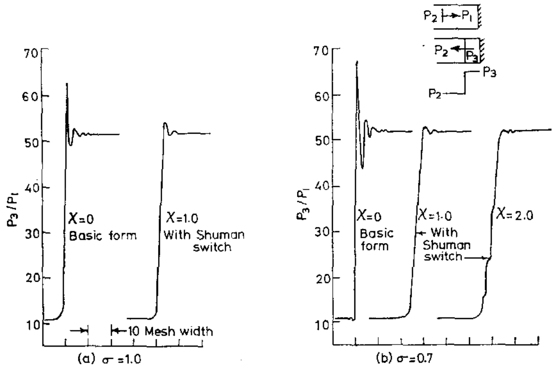


FIG. 5. Pressure profiles for the reflected shock as given by the second-order Richtmyer scheme with and without the Shuman Switch.

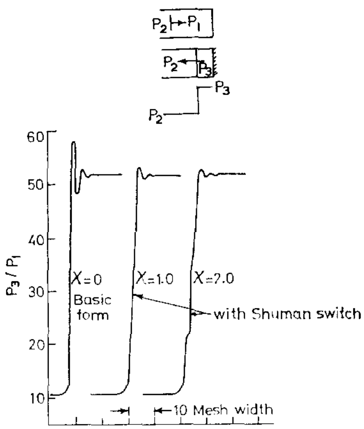


FIG. 6. Pressure profiles for the reflected shock as given by the second-order MacCormack scheme with and without the Shuman Switch.

felt for $\chi = 2.0$ where the profile is rendered sharper especially at the upstream of the propagating shock. The exponential form with $\chi = 2.0$, $m = 2.0$ again gives rise to a cascadelike structure for both the propagating and the reflected shocks. The above conclusions are true even for the shock in the shock tube flow (Figs. 11–14). The weak shock here is smeared to about 10 mesh widths. The oscillations near the contact discontinuity are not eliminated. An elimination of these is possible only with certain disadvantages like smearing and presence of a large undershoot (Fig. 12). Figure 13 shows that the various improved forms of

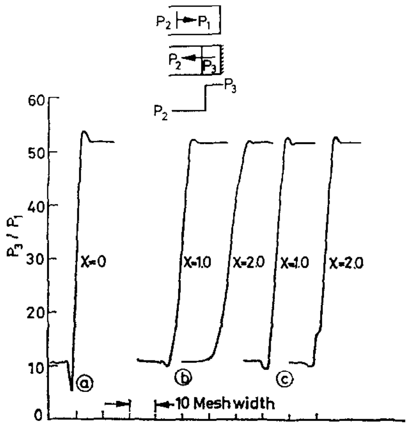


FIG. 7. Pressure profiles for the reflected shock as given by the third-order Rusanov scheme: (a) basic form $\sigma = 0.7$, (b) with the Shuman Switch in basic form, $\omega = 2/24$, $\sigma = 0.7$, (c) with the Shuman Switch in exponential form, $\omega = 2/24$, $\sigma = 0.7$.

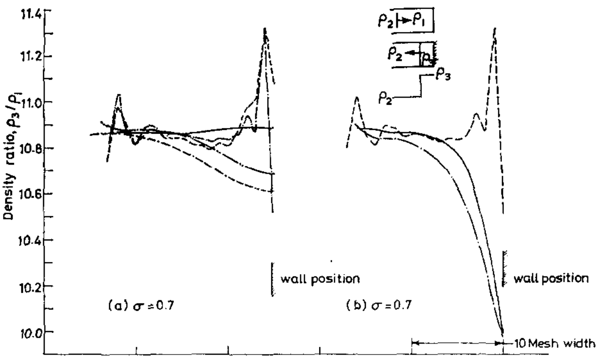


FIG. 8. Density profiles near the wall (enlarged) as given by the various schemes for the shock reflection problem: (a) – – – · – – – · first-order Rusanov scheme, $\omega = 1.0$, —— · · —— · · first-order Van Leer scheme, $\omega = 1.0$, ——·——·—— second-order Richtmyer scheme, – – – – second-order MacCormack scheme, —— third-order Rusanov scheme; (b) – – – Richtmyer scheme without the Shuman Switch. —— Richtmyer scheme with the Shuman Switch, $\chi = 1.0$, ——·——· Richtmyer scheme with the Shuman Switch, $\chi = 2.0$.

the switch do not appear to offer any significant improvement in the behavior of schemes. The Shuman Switch was employed in the Richtmyer scheme for the example of the standing shock. The instability was overcome and stable results were obtained for $\chi = 1.0$ and $\chi = 2.0$. Large smearing even for $\chi = 1.0$, is evident from Fig. 9.

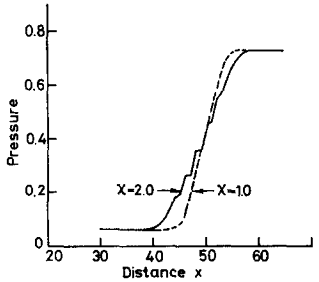


FIG. 9. Pressure profiles for the standing shock as given by the Richtmyer scheme with the Shuman Switch ($\sigma = 0.7$).

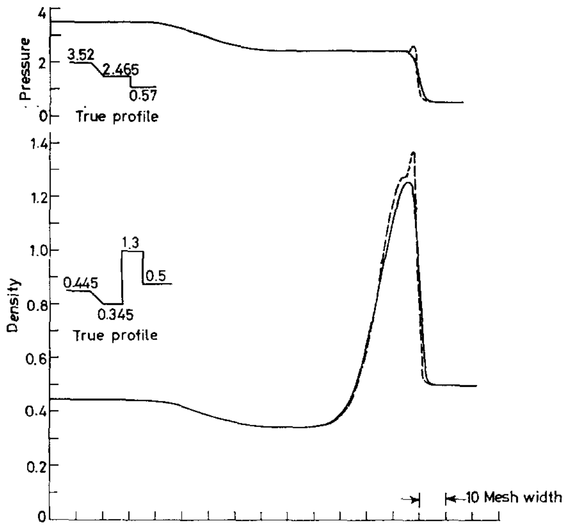


FIG. 10. Density and pressure profiles for the shock tube flow as given by the first-order schemes. $\sigma = 0.7$. —— the Rusanov scheme, - - - the Van Leer scheme.

(iv) *Boundary Errors for Different Schemes*

It is noticed that (Fig. 8) with the exception of the third-order scheme and the MacCormack scheme, all other schemes considered predict a lower value of density at the wall. The MacCormack scheme predicts a higher value. The manner in which the density profile joins the theoretical value behind the reflected shock is seen to be a function of the order of the scheme. The prediction of a lower value of density at the wall could be accounted for by following the arguments advanced by Fox [15]. When the shock interacts with the wall and the reflected shock is formed, there is an over production of entropy and subsequent heating at the wall.

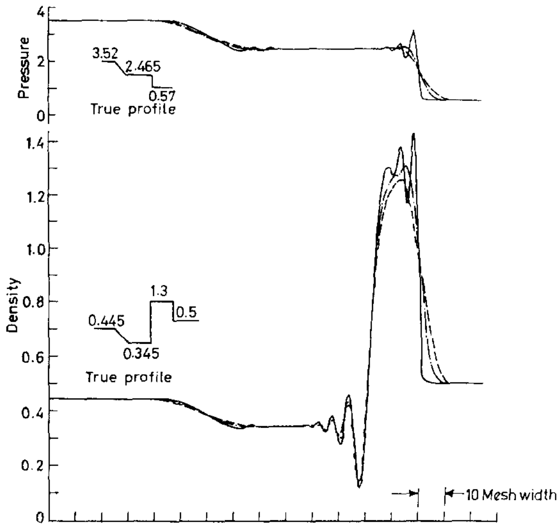


FIG. 11. Density and pressure profiles for the shock tube flow as given by the second-order Richtmyer scheme $\sigma = 0.7$, —— Basic scheme, —·—·— with the Shuman Switch $\chi = 1.0$, - - - with the Shuman Switch, $\chi = 2.0$.

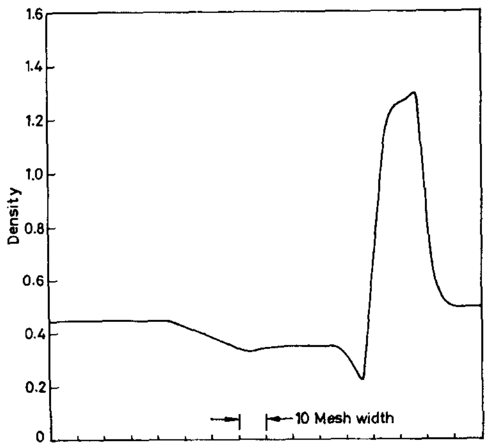


FIG. 12. Density profile as given by the Richtmyer scheme with the Shuman Switch ($\chi = 2.0$) oscillations near contact discontinuity eliminated.

This results in the lowering of the density value and thus large error in density value accumulates at the wall. The first-order schemes tend to be diffusive even in presence of a zero-velocity field (which exists between the wall and the reflected shock) and hence the above error gets diffused over a number of mesh widths in

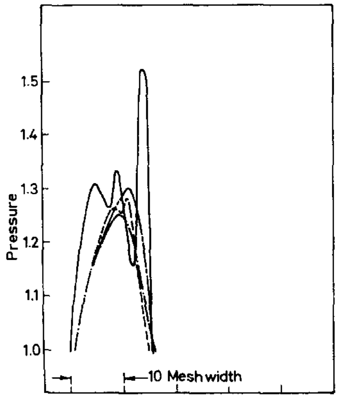


FIG. 13. Enlarged density profiles, between contact discontinuity and shock, as given by the Richtmyer scheme with the various forms of Shuman Switch ($\chi = 2.0$), —— without the Shuman Switch, —·—·— switch in the basic form, $m = 1.0$ — — — — — switch with $m = 2.0$, - - - - switch in the exponential form, $m = 2.0$.

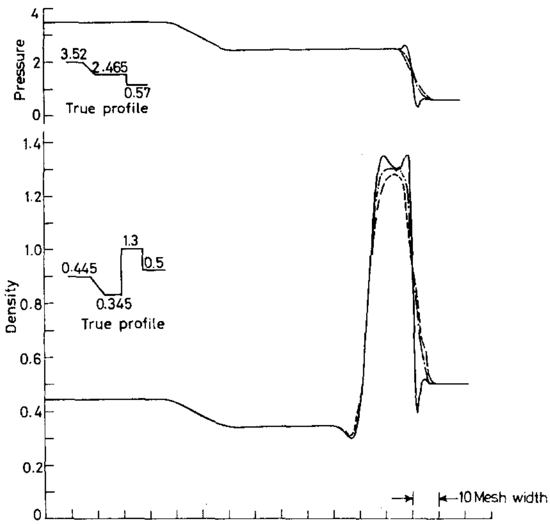


FIG. 14. Density and pressure profiles for the shock tube flow as given by the third-order Rusanov scheme, $\omega = 2.0/2.4$, $\sigma = 0.7$, —— basic scheme, —·—·— with the Shuman Switch, $\chi = 1.0$, - - - - with the Shuman Switch, $\chi = 2.0$.

the course of time. Similar reasoning holds good when the Shuman Switch is employed with the higher order schemes, where the order of accuracy is lowered to one wherever the switch is "ON." By the suitable choice of a shock-sensing property it is possible to avoid the artificial smearing at the wall. But in this case one has to accept the artificial oscillatory profile given by the basic second-order scheme.

(v) *Behavior of Schemes for the Standing Shock*

The first- and the third-order methods, as expected, gave stable results for the standing shock problem. The smearing, overshoot and other features of the profiles were similar to those observed for the example of the propagating shock. The Richtmyer scheme was found to be unstable. The MacCormack scheme exhibited a mild instability allowing the computations to proceed. But the profile was unacceptable. This nonlinear instability can be overcome by employing the Shuman Switch. This has been demonstrated for the Richtmyer scheme in Fig. 9. Further discussion regarding this may be found in Section iii.

Concluding Remarks

With reference to the questions which were posed earlier the following conclusions can be drawn.

1. Compared to the Rusanov scheme, the Van Leer scheme does give sharper shock profiles which also are smooth for values of σ around 0.7. The improvement regarding other discontinuities is marginal.
2. The MacCormack scheme is seen to behave better than the Richtmyer scheme in all aspects of shock handling.
3. The Shuman Switch is effective in overcoming oscillations and instabilities but considerably smears the shock and the weak shocks are smeared to an intolerable extent.
4. The first- and the second-order methods both give rise to artificial heating at the wall and hence artificial density profile near the wall in the case of shock-wave reflection.
5. The nonlinear instability associated with the standing shock can be overcome by the use of the Shuman Switch provided one can accept the largely smeared shock profile.

When we consider the results of the present work together with those of earlier investigations (Table I), it is observed that no single scheme performs best in every aspect considered. The recommendation by Taylor *et al.* [3] of Godunov's scheme [14] (not tested here) is, of course, unaffected. The Godunov scheme, however, requires considerable programming effort and computer time, while the simplified version given by Van Leer [13] again requires extra viscosity terms to overcome nonlinear instabilities. For those who want a first-order scheme as simple as Rusanov's, but with a somewhat higher resolving power, the Van Leer scheme with quadratic diffusion coefficients may be an alternate choice. The reduction in diffusion, however, is paid with an increase in overshoot.

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